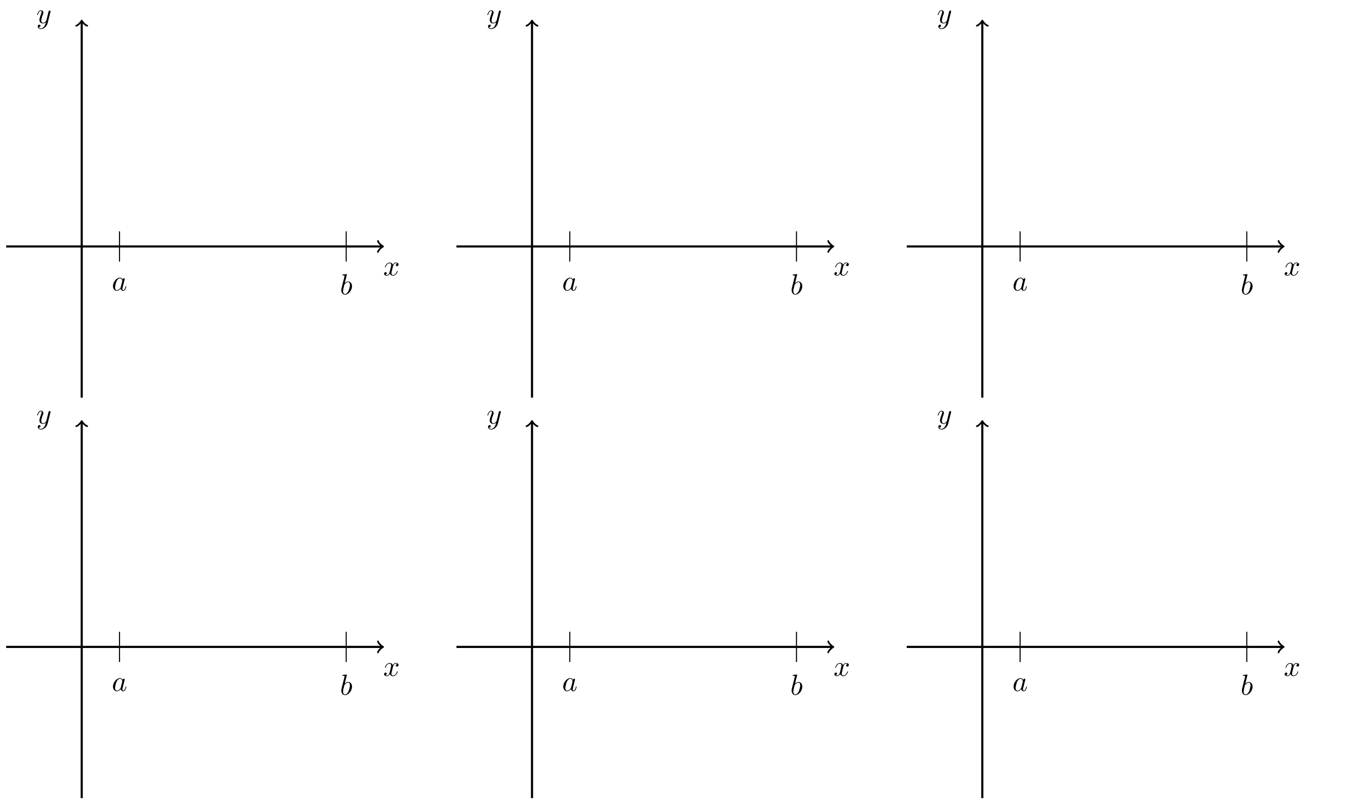
LECTURE NOTES: 4-2 THE MEAN VALUE THEOREM (PART 1)

MOTIVATING EXAMPLES: Draw several examples of graphs of functions such that (i) the domain is $[a, b]$ and (ii) $f(a) = f(b)$. Note you are not *required* to make sketches that are continuous or differentiable, though you may choose to do so.



QUESTION 1: What does it mean to call something a *Theorem* in a mathematics course?

QUESTION 2: What is the difference between a *conjecture* and a *Theorem* in a mathematics course?

$\boxed{\text{QUESTION 3:}}$ State in plain old English (or draw a picture) to explain what it means for the graph of $f(x)$ if you know $f'(c) = 0$.

$\boxed{\text{QUESTION 4:}}$ Based on our examples on the previous page and your knowledge of graphs more broadly, what requirements would be needed to *guarantee* the existence of an x-value c in the open interval (a, b) such that $f'(c) = 0$?

$\boxed{\text{ROLLE'S THEOREM:}}$ If ,

then there is a number c in the interval (a, b) such that $f'(c) = 0$.

$\boxed{\text{QUESTION 5:}}$ Now that we see a pattern, can we give an argument for why that pattern should hold? (HINT: What does the Extreme Value Theorem say again??)

PRACTICE PROBLEMS:

1. Consider $f(x) = x^3 - 2x^2 - 4x + 2$ on the interval $[-2, 2]$.

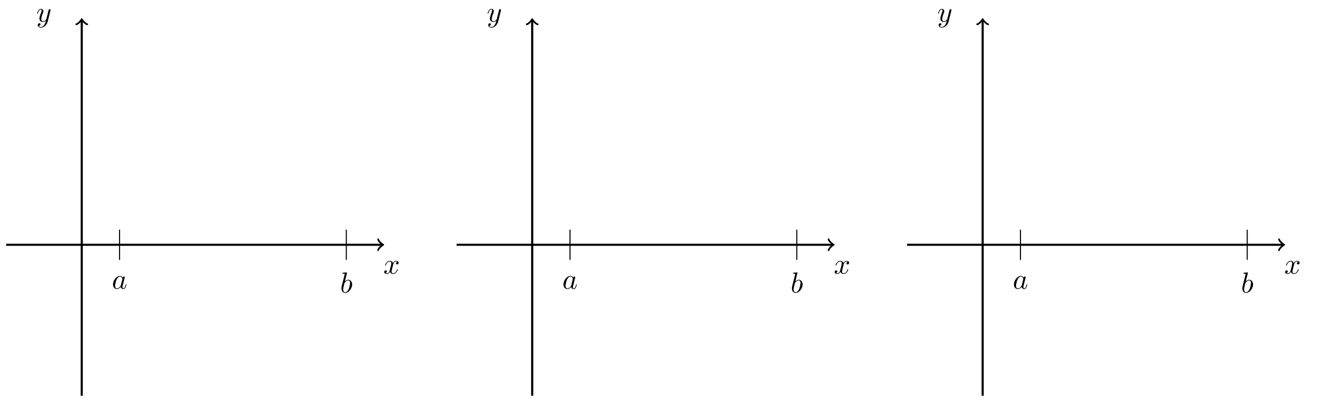
   (a) Verify that the function $f(x)$ satisfies the hypothesis of Rolle's Theorem on the given interval.

   (b) Find all numbers c that satisfy the conclusion of Rolle's Theorem.

   (c) Sketch the graph on your calculator to show that your answer above are correct.

2. *Use Rolle's Theorem* to show that the equation $x^3 - 15x + d = 0$ can have at most one solution in the interval $[-2, 2]$.
   HINT: Show that there is no way there could be two solutions!

MOTIVATING EXAMPLES: Draw several examples of graphs of functions such that (i) the domain is $[a, b]$, (ii) $f(x)$ is continuous on $[a, b]$, **and** (iii) $f(x)$ is differentiable on $[a, b]$. We are *not* assuming that $f(a) = f(b)$.



QUESTION 6: In each picture above, draw (or in some other way identify) the quantity:

$$\frac{f(b) - f(a)}{b - a}.$$

What would this quantity be if Rolle's Theorem applied?

THE MEAN VALUE THEOREM: If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b), then there is a number c in the interval (a, b) such that

OBSERVATION: The Mean Value Theorem is just Rolle's Theorem if you turn your head sideways.

QUESTION 7: Assume that $f(x)$ is continuous and differentiable on the interval $[a, b]$ and assume there exists some x-value d in (a, b) such that $f(d) > f(a)$, can you draw any conclusion about $f'(x)$? Why or why not?

THEOREM 5: If $f'(x) = 0$ for all x in the interval (a, b), then

$$\boxed{}.$$

QUESTION 8: How would you explain why this theorem is true? (Hint: See your answer to Question 7!)

QUESTION 9: If $f(x)$ gives the *position* of an object as a function of time, what "common sense" idea is the MVT telling us? Theorem 5?